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SYMMETRY IN WORLD TRADE NETWORK*

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Abstract Symmetry of the world trade network provides a novel perspective to understand the world-wide trading system. However, symmetry in the world trade network (WTN) has been rarely studied so far. In this paper, the authors systematically explore the symmetry in WTN. The authors construct WTN in 2005 and explore the size and structure of its automorphism group, through which the authors find that WTN is symmetric, particularly, locally symmetric to a certain degree. Furthermore, the authors work out the symmetric motifs of WTN and investigate the structure and function of the symmetric motifs, coming to the conclusion that local symmetry will have great effect on the stability of the WTN and that continuous symmetry-breakings will generate complexity and diversity of the trade network. Finally, utilizing the local symmetry of the network, the authors work out the quotient of WTN, which is the structural skeleton dominating stability and evolution of WTN.

Key words Automorphism group, network quotients, symmetry, world trade network.

1 Introduction

Symmetry in physics has been generalized to characterize invariance, that is, lack of any visible change-under any kind of transformations[1]. Symmetry is universal in life, e.g., bodies of mankind and animals, the buildings of different countries and various vases, and so on. Symmetry is one of the most powerful tools of theoretical physics, chemistry, and life science[2–3]. Symmetry is closely related to harmony, beauty, and unity, which determines that it will play an important role in theories of nature[4]. For example, symmetry has been found to be of great practical use in simplifying complex problems[5]. Symmetry also can be considered as a significant property to describe the state of the system. Consequently, it is necessary to analyze symmetry in the system when exploring static or dynamic properties of systems[6].

Researches on complex networks have attracted widespread attention in recent years, and have made great progress[7–10]. However, an important property of network structure, symmetry has been rarely studied in the community of complex networks[11–17]. In the paradigms of physical studies, exploring symmetry implicated in the system is one of the most important methods to find elementary laws dominating the dynamics of various systems. Actually, symmetry will have a profound effect on the robustness and dynamic evolution of the network structure[18–20].

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Similar to concepts of other symmetries in physics, the symmetry of network structure characterizes the invariance under certain transformations. However, the detailed definition of symmetry in network structure is different from that of other symmetries in physics in the meaning of "invariance" and "transformation". Specifically, symmetry in network structure characterizes the invariance of adjacency of vertices under the permutations on vertex set, which implies that "invariance" of the symmetry in network structure is the relation among vertices and the "transformation" is permutations on vertex set[31].

The distinctive characteristic of symmetry in network structure is determined by the essence of networks, i.e., relations among individual components in the network. Recall that the most important information conveyed by a network is the relation among the vertices in the network, which determines that the "invariance" of the symmetry of the network structure is the relation among the vertices. The fundamental constituents of a network are the individual components or vertices; thus, the permutations on these vertices will be the most important transformation on networks. Under these transformations, whether the relations among the vertices are preserved critically indicates whether the underlying properties of the network are preserved or not. Therefore, whether the relations of the vertices are preserved or not under the transformations of vertex set underlies the researches on symmetry of network structure.

Recently, there has been an increasing recognition to study the world trade from a network perspective and preliminary achievements have been made. For example, the topological structure, scale-free property, complexity and synchronization of the world trade network (WTN) have been studied[22−26]. However, few works have been dedicated to the research on the structural symmetry of WTN.

Deep insight into the symmetry property of WTN is beneficial for us to understand the import/export trade relations among different countries deeply, so as to improve the future development of world trade. Therefore, exploring the symmetry in WTN will be a fascinating and important problem and the work in this paper is devoted to studying this problem.

The structure of this paper is as follows. In Section 2, essential concepts about network symmetry are introduced. In Section 3, we construct the world trade network based on the world import/export trade data in 2005. We find that WTN is symmetric, more precisely, local symmetric to a certain degree by exploring the size and structure of its automorphism group. Furthermore, we factorize the automorphism groups of WTN and work out its symmetric motifs. Through analyzing the structure and function of symmetric motifs, we find that local symmetry in WTN have great effect on the dynamic stability of WTN, particularly, the emergence of complexity and diversity can be attributed to the continuous symmetry-breakings of its local symmetric substructure. In Section 4, we work out the quotient network of WTN by removing the redundancy hidden in the network symmetry, which is the structural skeleton dominating stability and complexity of WTN, thus ultimately determining the further development of WTN. Finally, we finish the paper with some general conclusions.

2 Preliminaries

2.1 Automorphism Groups

The automorphism groups are the elementary tools to explore symmetry in the network structure. In this section, we first give some preliminaries related to automorphism group of a graph.

A graph or network is denoted by $G = G(V, E)$, where V is the vertex set, E is the edge set. A mapping from vertex set V to itself is a permutation if it is injective and surjective. We use the notation Ω for the set of all permutations acting on V. Let $*$ be a binary operation on Ω, which is defined as $\forall p, q \in \Omega, v \in V, (p * q)v = p(qv)$. An automorphism of a graph G is a permutation π of the vertices of the network that preserves adjacency of the vertex set V, i.e., any vertices v_1 and v_2 are adjacent if and only if πv_1 and πv_2 are adjacent. The set of automorphisms of the graph under the permutation operations forms a group[34], which is denoted as $\mathrm{Aut}(G)$, and the size of which is α_G. The fact that $\mathrm{Aut}(G)$ is a group means that 1) $\forall a, b \in \mathrm{Aut}(G)$, there exists a unique element $c = a * b \in \mathrm{Aut}(G)$; 2) for all $a, b, c \in \mathrm{Aut}(G), a * (b * c) = (a * b) * c \in \mathrm{Aut}(G)$; 3) there exists a unique element $e \in \mathrm{Aut}(G)$ such that for all $a \in \mathrm{Aut}(G)$, $a * e = e * a = a$; 4) for every element $a \in \mathrm{Aut}(G)$, there exists an inverse element $a^{-1} \in \mathrm{Aut}(G)$ such that $a * a^{-1} = a^{-1} * a = e$.

Symmetry in network structure is usually measured by the size of the automorphism group. A network is said to be symmetric (respectively asymmetric) if its underlying graph has a nontrivial (respectively trivial) automorphism group. Usually, the degree of symmetry of a network is quantified by α_G. In order to compare networks of different sizes with each other, the quantity $\gamma_G = (\alpha_G/N_G!)^{1/N_G}$ is also usually used[21], which measures symmetry relative to maximum possible symmetry (the complete graph on N_G vertices and its complement–the empty graph on vertices, are the most symmetric graphs, both having $\alpha_G = N_G!$). We illustrate all these concepts through the following example.

Example 1 Figure 1 shows two symmetric graphs; the sizes of the automorphism groups of G_1 and G_2 are $|\mathrm{Aut}(G_1)| = 72$, $|\mathrm{Aut}(G_2)| = 12$, respectively. A set of generators[30] of graph G_1 is $\{(2,3),(7,8),(6,7),(1,2),(1,6),(2,7),(3,8),(4,5)\}$ and a set of generators of graph G_2 is $\{(2,3),(6,7),(1,2)\}$.

2.2 Local Symmetry

Although any automorphism can preserve adjacency among vertices, different automorphisms act on the set of vertex in different ways. In the graph shown in Figure 1, the automorphism $(1,2)$ just needs to transform vertices 1, 2 to each other to preserve adjacency among all vertices. However, for the automorphism $(1,6)(2,7)(3,8)(4,5)$, to preserve the adjacency of the whole vertex set, all vertices need to be mapped to a different one. Intuitively, the latter represents the structural symmetry in a global sense, while the former represents the structural symmetry in a local sense. Distinguishing between global symmetry and local symmetry is helpful to gain a deeper insight into the self-organizational principles of the network.

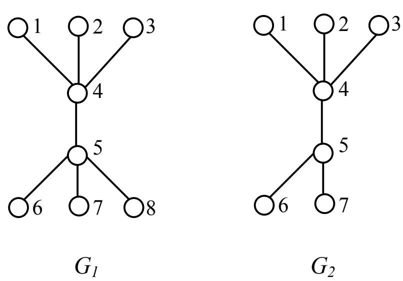


Figure 1 Illustration of symmetry in graph

However, it is nearly impossible for real-world networks to be globally symmetric[17]. In other words, real networks have less chance to have automorphisms that can move all vertices. Because permutations of "global" symmetries move distant vertices while preserving adjacency, such symmetries are sensitive to changes in network, implying that even small perturbations may result in symmetry breaking of the network structure. For instance, the size of automorphism groups reduce substantially from 72 (graph G_1) to 12 (graph G_2) after vertex 8 in the graph

G_1 has been removed. In this case, a large number of global symmetries have been broken, which result in the reduction of size of the automorphism group. Therefore, global symmetry in networks suggests that strong organizational principle dominates the formation of network structure, which is not probably present in most real-world networks. Therefore, here we only explore the local symmetry in WTN, highlighting the fact that networks in the real-world are generally local symmetric.

The concept of local symmetry is based on permutation group theory, which is defined as follows: A network G is locally symmetric if $\mathrm{Aut}(G)$ can be factorized into a large number of geometric factors:

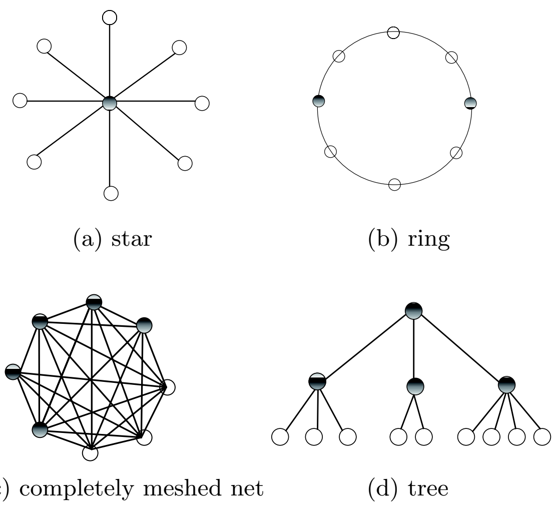
$$\mathrm{Aut}(G) = H_1 \times H_2 \times \cdots \times H_n,$$

where each factor acts locally on the network[17].

The automorphism group is of great practical use, since we can construct symmetric motifs by utilizing its geometric factors. Information extracted from symmetric motifs will be valuable to explore the microscopic mechanisms accounting for the emergence of symmetry in the network.

We shall call the induced subgraph on the support of a geometric factor H a symmetric motif[17], and the induced subgraph on a set of vertices $V_0 \in V(G)$ is the graph obtained by taking V_0 and any edges whose end points are both in V_0.

Symmetric motifs may have various structures, such as ring, star, tree, and so on (shown in Figure 2). In the graphs of Figure 2, the solid vertices are the symmetric center of the symmetric motifs, which are significant for various properties of networks, and usually their vertex degree are larger than hollow ones. Usually, vertices in the symmetric motif can only communicate with the outside world through the symmetric center, due to the fact that the hollow vertices are only adjacent to the symmetric center in most symmetric motifs. These two kinds of vertices have different roles and functions, exerting different influences on the property of network systems.



(a) star (b) ring

(c) completely meshed net (d) tree

Figure 2 Symmetric motifs with different structures

2.3 Network Quotients

Given the automorphism group acting on vertex set V, we can establish an equivalent relation on the vertex set, which is defined in the way that vertex x is equivalent to vertex y if and only if there exists $g \in \mathrm{Aut}(G)$, s.t. $gx = y$. According to this equivalent relation on vertex sets, we can construct a corresponding partition on the vertex set, denoted as $\Delta = \{\delta_1, \delta_2, \cdots, \delta_n\}$, which is usually called as automorphism partition on vertex set. Each cell of the automorphism partition is called as an orbit[34] of $\mathrm{Aut}(G)$; thus, automorphism partition also can be called as a system of orbits of a graph. The fact that the vertex set of a network can be partitioned into the system of orbits helps us to solve two kinds of problems: 1) Evaluation of equivalent relation among the vertices. 2) Simplification of the network structure.

From the network structure perspective, it is effective by utilizing the orbit system to explore the equivalent relation among the vertices. According to the concept of orbit, the vertices in the same orbit will play the same role in network structure, which means that these vertices will have the same properties or the value under the measurement of certain well-defined structural measures on individual vertices[33]. These measures can be characterized as orbit conserved. Formally, a measure on the vertices of G, $m(v)$ is orbit conserved if $m(v) = m(\pi(v))$ for $\pi \in \mathrm{Aut}(G)$. It has been shown that vertex degree[15], eigenvector centrality[22], and clustering coefficient[22] are all orbit conserved.

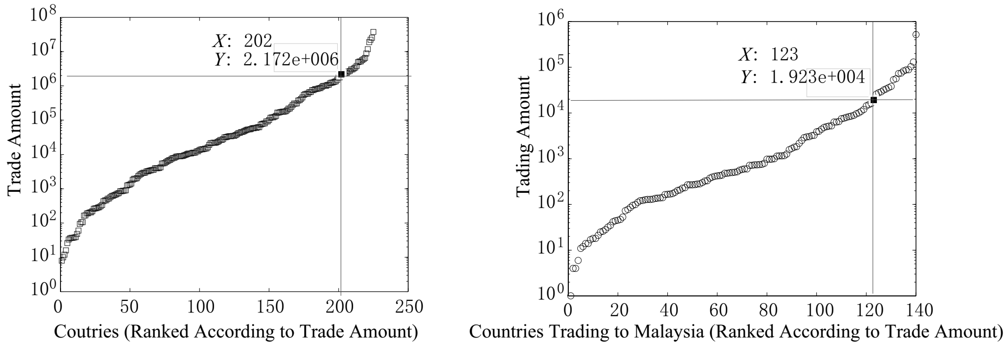
Utilizing the equivalent relation on the vertex set, we can simplify the network structure through abstracting the vertices in the same orbit into one vertex, which has been systematically investigated in [32]. Then, for orbit conserved measures, e.g., the measures mentioned above, we only need to calculate the value of these measures for each orbit instead of every vertex in the network. In practice, this means that rather than working with the full network, we can calculate many network properties via the quotient. The definition of the quotient of the network is as follows. Let d_{ij} be the number of edges starting from any vertex in δ_i and ending in vertices in δ_j. Since the orbits partition the vertex set into disjoint equivalence classes, d_{ij} depends on i and j alone. The quotient under the action of $\mathrm{Aut}(G)$, denoted by $\varphi = \frac{G}{\mathrm{Aut}(G)}$, is the multi-digraph with vertex set Δ and adjacency matrix $[d_{ij}]$.

3 World Trade Network

3.1 Construction of World Trade Network

Mathematically, the world trade network is a graph, denoted as $G = G(V, E)$, with vertex set V representing countries and edge set E representing the trade relations among these countries. The number of vertex set and edge set are denoted by N_G and M_G, respectively. The trade relations among the countries are defined as edges, i.e., there exists an edge between two vertices if trade relation is established between corresponding two countries. Two vertices are said to be adjacent if there is an edge between them.

We collected world trade data about 225 developed or developing countries and areas in year 2005 to construct the world trade network. As is evident in Figure 3(a), the distribution of the world trading volume is not uniform, and nearly 85% of trading volume concentrates on 24 countries and areas (such as China, USA, UK, French, and so on). Moreover, they trade with almost all the countries in the world, so we call them the cores of the trade network. From Figure 3(b), it can be observed that for every single country, the distribution of trading volume is also non-uniform (close to exponent distribution). Therefore, for each non-core country, we keep part of its whole trade relations, such that the sum of the trading volume of these kept trade relations is up to 85% of the total trading volume of this country. Thus, most of the trading amount of these non-core countries will be preserved, and the whole trade network constructed under this principle can capture most of the trade amount of the entire network. Furthermore, we removed the relatively unimportant edges (the trade volume of these edges are relatively small to the kept, nearly can be neglected) to make the resulting network more



(a) Distribution of trading volume of the world (b) Distribution of Malaysian trading volume

Figure 3 Distribution of trading volume

meaningful in real applications. After that, we use the underlying graph of the network, i.e., we remove weights and directions from the network.

After pre-processing following the above principle, there remains overall 225 vertices and 971 edges in the resulting network, which is illustrated in Figure 4 (The network is visualized by Pajek program[27]).

3.2 Local Symmetry of the World Trade Network

Using nauty algorithm[29], we calculate the size of the automorphism group of WTN and an ER random network with the same size as the WTN. The results are shown in Table 1. The size of the automorphism group (α_G) of WTN is 5.0617×10^{22} and the degree of relative symmetry (β_G) is 0.0149, suggesting that the WTN is symmetric to a certain extent. The size of the automorphism group of the corresponding ER network only equals to 1, i.e., $\alpha_G = 1$, which conforms to the argument in classical ER theories that large ER random network is asymmetric[21]. The fact that the symmetry in real world trade network is more abundant than that in the corresponding ER random network with the same size suggests that symmetry may be related to certain self-organizational principles of networks.

Table 1 Optimal starting price and resulting expected revenue for each stage

Network	N_G	M_G	z (average degree)	α_G	γ_G
WTN	225	971	4.3155	5.0617×10^{22}	0.0149
ER	225	935	4.3155	1	0.0119

Based on the geometric factor decomposition of the automorphism group of WTN, we obtain all symmetric motifs in WTN, some of which are shown in Figure 5.

3.3 The Meanings of Local Symmetry in the WTN

A symmetric motif represents a certain local area of the WTN, with solid vertex representing the trading center of the area. The healthy status of the trading center will have great influences on the local area. It is the trading center in which the local trading markets centralize, that connects the markets inside the local trading area with the outside. Therefore, prosperity of the trading center will accelerate the development of the whole local trading area. On the contrary, if the trading center suffers from economic depression due to the exogenous shock or its own instability, the local trading area will be subjected to bad consequences. Since foreign trade is an important channel for economic interactions with different countries, a crisis breaking out in the trading center will propagate to other countries rapidly and result in unsteady economic environment in the local trading area. The key is that most of the countries in the local trading area trade with outside markets only through the trading center. As a result, once the trading center suffers from any severe damage, the local trading area may break away from the WTN. Consequently, the structural stability of the WTN will be broken seriously.

Figure 4 The world trade network (Data Source see [28])

The hollow vertices in the symmetric motif, i.e., non-central countries in the local trading area are very dependent on the trading centers. Trading with the local trading center is usually their critical path to expand the technical and economic exchange, and to construct a favorable external environment for domestic development. However, we can see that in many symmetric motifs in the WTN, especially in the symmetric motifs in the form of a star (as shown in Fig (j)), the trading center is not only the most important trading partner for other countries in the local area, but also is the unique trading partner, so that the trade development in these countries is limited greatly to the local area. This observation implies that these non-central countries may undergo decays when the trading center suffers from crisis. Once the trading relation between hollow vertices and the central vertex is damaged, trading volume of the countries represented by hollow vertices will go down substantially, and will consequently have negative effect on their domestic trade development. What's more, it is possible for those hollow vertices in the star symmetric motifs to break away from the trading center, and then fall into the predicament like the isolation ward. In the worst case, the butterfly effect will arise, so that the stability of the trade in the entirely local area will be affected.

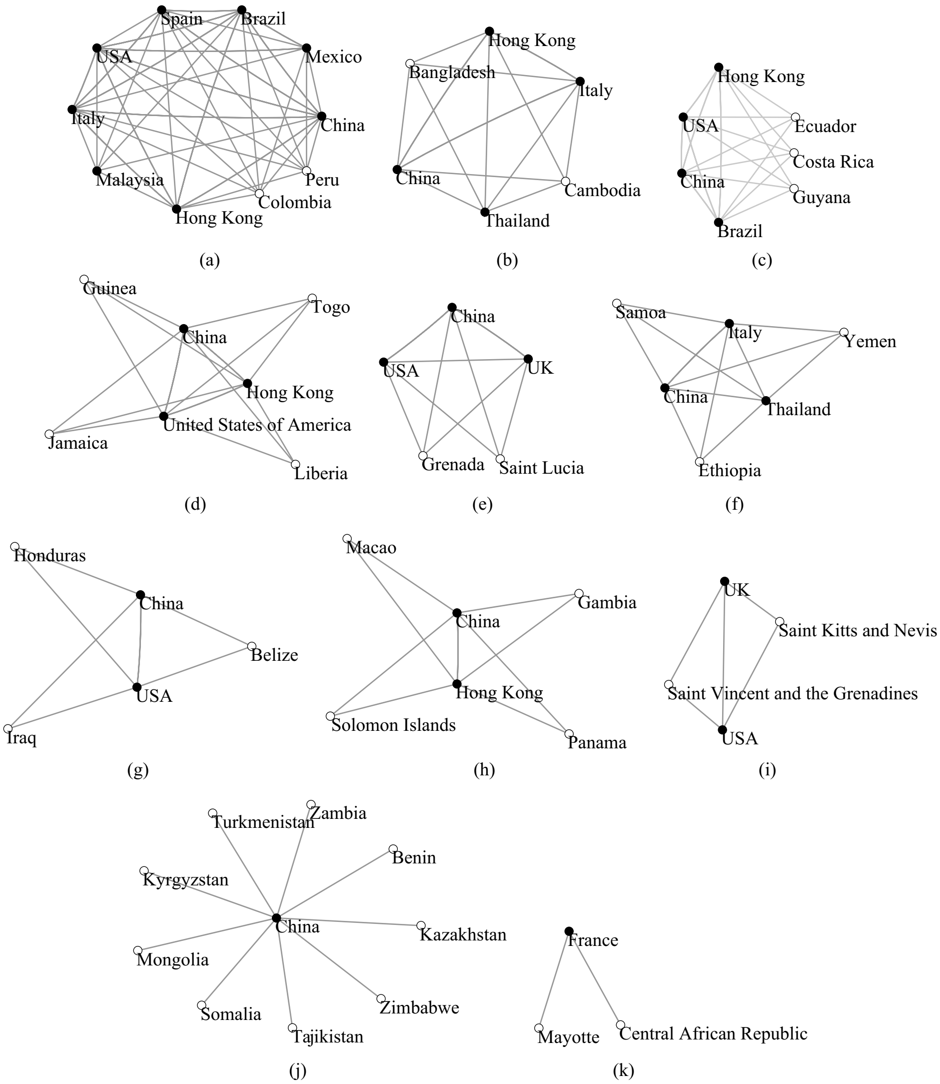


Figure 5 Symmetric motifs in WTN

Therefore, multilateral trade relations should be established as much as possible for any country, especially, for countries represented by hollow vertices in the star symmetric motif, to expand space for domestic trade development, improve abilities to resist exogenous shock and risks. Hence, only when multilateral trade relations has been built, the stability of the trade network will be improved.

Since the countries that are structurally equivalent in a symmetric motif have the same structural properties in the corresponding trading area, it is reasonable to conjecture that they tend to have the similar behavior patterns, such as the trading behaviors and risk-resisting behaviors. Hence, it is not surprising to find that these countries tend to learn from each other in the formation of the trade policies to speed up domestic trading development. Furthermore, since the trading relations of these countries are usually limited in the local trading area, it is also very possible for these countries to compete with each other. Thus, once economic crisis breaks out in one country, the role of this country in the local market will probably be replaced by another structurally equivalent country in the same local area.

Figure 6 The network quotient of the world trade network

The evolution of local symmetric motifs has a profound effect on the evolution of the whole structure of WTN. Local symmetries of the trade network can be broken by the change of trading relations in the local area, which is caused by the continuous developments of the foreign trade of different countries. Also, we believe that it is just the symmetry breakings, specifically local symmetry breakings that is responsible for the emergence of complexity and diversity of the whole trade network.

3.4 The Trade Network Quotient and Its Implications

We abstract the vertices in the same orbit of the WTN into one vertex and get a simplified network (please refer to [32] for the detailed calculation procedure), which is called as the quotient network of its parent network. The quotient network of WTN is shown in Figure 6. There are 179 vertices and 881 edges in the quotient network. The number of vertex is 79%

of that of original WTN and the number of edges is 90% of that of original WTN. Compared to the original trade network, the size and complexity of the trade network quotient are all reduced.

From the symmetry perspective, the quotient network strip all symmetric elements from the original WTN, and consequently, improving asymmetry and order of the network. Since elements contributing to the simplicity or redundancy have been abstracted in the quotient network of the WTN, what remained in the quotient network is just the skeleton, which is the core of the original network consisting of the elements contributing to the complexity or heterogeneity of the network system[32]. Thus, it is reasonable to believe that it is the quotient network of the WTN that determines the essence of the network structure, thus dominating the static properties and dynamic evolution of WTN.

4 Conclusions

We have constructed the world trade network and investigated the size and structure of its automorphism group. As a result, we find that a certain degree of symmetry, specifically, local symmetry exists in the WTN. We have worked out the symmetric motifs of the WTN. Each symmetric motif is associated with a local trading area of the WTN. We have shown that these symmetric motifs will have great effect on the stability of the WTN. Finally, we worked out the quotient network of WTN, which is the structural skeleton of WTN.

Symmetry and asymmetry of the WTN change continuously with the evolution of WTN. However, the underlying mechanisms responsible for the emergence of symmetries and symmetry breakings are still unknown. With the economic globalization and liberalization increasing, trade relations among different countries will be more frequent and close. As a result, the dynamic evolutional process of WTN will be more complicated. Therefore, symmetry and symmetry breaking will play more important roles in researches on WTN.

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